GRAVITATIONAL WAVE GENERATION FROM COSMOLOGICAL PHASE TRANSITIONS

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3^o Edition of the Friends-of-Friends Meeting
2013 April

Outline

What are we interested in?

Particle Physics predicts several phase transitions:

- Grand Unification (GUT) ($T \sim 10^{15}$GeV)
- **Electroweak Transition** ($T \sim 100$GeV)
- Quark-Hadron Transition (QCD) ($T \sim 100$MeV)

(We use *natural units*: $c = \hbar = k_B = 1$ and eV $\sim 10^4$K)

This talk is focused on the electroweak phase transition, which has several potentially observable cosmological consequences:

- Magnetic fields
- Baryogenesis, baryonic inhomogeneities
- **Gravitational Waves** (This is what interests us!)

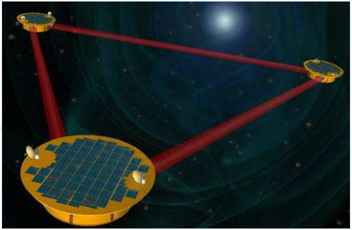
Note:

Most of the concepts that will be discussed here are valid for other phase transitions.

Gravitational Wave generation *in a phase transition*

Bulk motions of the relativistic fluid in a phase transition are a source of gravitational waves.

- Unlike electromagnetic radiation, gravitational radiation, once generated, propagates freely.
  - It could provide information about earlier stages of the universe.
- The frequency of gravitational waves generated in the electroweak phase transition would be $\sim$ miliHertz.
  - It lies within LISA's sensibility range.



Laser Interferometer
Space Antenna

Characteristic frequency and amplitude

The spectrum essentially depends on a few parameters related with the dynamics of the transition

- The peak wavelength of the gravitational radiation is determined by the characteristic length of the source , $\lambda_p \sim L_S$
- Then, peak frequency is given by $f_p \sim 1/L_S$
- Once generation occurs, the waves propagate freely until present
- Taking into account the redshift, the peak frenquency today is given by

$$f_p \sim \frac{1}{L_S}\left(\frac{a_*}{a_0}\right) \sim 10^{-5}\,\mathrm{Hz}\,\frac{T}{100\,\mathrm{GeV}}\,\frac{H_*^{-1}}{L_S}.$$

  - a_* and a_0 are the scale factor at that moment and today.
  - $100\,\mathrm{GeV}$ = electroweak scale
  - H_*^{-1} = Hubble length at that moment ($H = \dot{a}/a$ *Hubble rate*)
  - (Here, we used adiabatic expansion and a Hubble rate given by the Friedmann equation.)

- Considering a perturbation $h_{\mu\nu}$ of the *metric* $g_{\mu\nu}$, linearizing *Einstein equation* and keeping the relevant degrees of freedom, we can obtain a *wave* equation of the form: $\Box h \sim GT$
  - $\Box$ is a linear operator which contains second derivatives.
  - $T =$ A projection of the energy-momentum tensor of the source.
  - $G =$ Newton constant.
- The length scale of the source gives the variation scale, $\partial_\mu \sim \frac{1}{L_S}$
  We can estimate h dimensionally: $L_S{}^{-2} h \sim G\rho_K \ \Rightarrow h \sim G\rho_K L_S{}^2$
  - $\rho_K =$ kinetic energy density in bulk motions of fluid.
- The gravitational waves intensity is closely related to energy density: $\rho_{\mathrm{GW}}(\mathbf{x}, t) \sim \langle \partial_t h_{\mu\nu} \partial_t h^{\mu\nu} \rangle / G \sim (\partial_t h)^2 / G \sim (L_S{}^{-1} h)^2 / G$
- Then, the wave energy density is $\rho_{\mathrm{GW}} \sim G\rho_K{}^2 L_S{}^2$
- Taking into account the redshift, the energy density today is

$$\rho_{\mathrm{GW}} \sim (\rho_K/\rho_*)^2 \, (L_S H_*)^2 \, \rho_0$$

  - (using $H_*^2 \sim G\rho_*$, *Friedmann Equation*)
  - ρ_* total energy density at this moment
  - $\rho_0 = \rho_* \, (a_*/a_0)^4$ radiation energy density today

In a nutshell

- We are interested in gravitational wave generation from the electroweak phase transition.
- Expected signal:
  - $\rho_{\rm GW} \sim {\rho_\kappa}^2 {L_S}^2$
  - $f \sim \frac{T_*}{L_S}$

  (Peak value estimations)
- In order to estimate the relevant parameters,
  it is necessary to study the hydrodynamics involved.

Electroweak theory

- Part of the particle physics *Standard Model*.
- Unifies electromagnetic and weak nuclear interactions.
- Involves all *Standard Model* particles: quarks, leptons, fotons, Z & W bosons, and the Higgs field.
- The particle masses depend on the Higgs field.

What is "the electroweak phase transition"?

- Reminding thermodynamics:
  - Every phase transition is governed by a thermodynamic potential (free energy) which depends on an order parameter.
  - In different regions of the universe, the order parameter takes different values, corresponding to different free-energy minima.
- In the electroweak phase transition the order parameter is the background (thermodynamic average) of the Higgs field.
- During the transition there is a coexistence of two minima associated with two phases. Each phase is characterized by:
  - Radiation (massless particles) - dominates at high temperature.
  - Massive particles (due to Higgs field) - dominates at low temperature.

Bubble nucleation

Classification of phase transitions:

- **1st order**: Supercooling or superheating occurs, there is a latent heat (an energy that, when is released, can "shake" and reheat the medium).
- **Superior order**: There is not a latent heat (Very boring!).

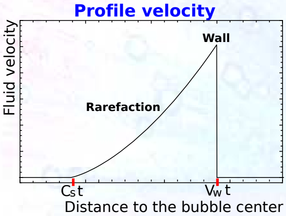
Characteristics of a strongly 1st order phase transition:

- Domains (bubbles) are nucleated containing the phase that is dominant at low temperature.
- Due to the supercooling there is a pressure difference between phases, which causes bubble expansion.
- The stronger the supercooling, the more violent the expansion and the stirring of the medium. This causes high-intensity gravitational waves.

Note 1: The 1st order transitions are not only interesting for us, also for those who study baryogenesis.

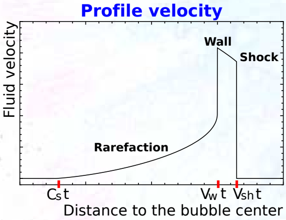
Note 2: Different extensions of the particle physics Standard Model may have different transition strength and hydrodynamics in the bubbles.

The bubble wall has two different propagation modes:



Detonation

- Supersonic wall (relativistic gas, $c_s \approx 1/\sqrt{3}$)
- Unperturbed fluid in front the wall
- *Rarefaction wave* following behind the wall



Deflagration

- Subsonic wall (respect to adjacent fluid)
- *Shock wave* preceding the wall
- Unperturbed fluid behind the wall (Usually) Although if the wall is supersonic, there is also a rarefaction "tail"

Turbulence in a phase transition

- The nucleated bubbles expand, either as detonations or deflagrations, dragging or pushing fluid until they collide generating turbulence.
- The generated eddies grow and also break into smaller eddies.
- So, there are different scales of eddies, which are source of gravitational waves with different wavelength.

Some calculation assumptions

This subject is complex and has several approximations...

- The phase transition is injecting energy at a single length scale, although there are bubbles with different sizes.
- There is some ambiguity to decide which scale is most relevant.
- In general, the literature in this subject doesn't consider the entire process from bubble collisions to the state of fully developed turbulence. (Great!, we have job yet)

Determination of hydrodynamic parameters

Remember these parametric dependencies:

- $\rho_{GW} \sim {\rho_K}^2 {L_S}^2$
- $f \sim \frac{T_*}{L_S}$

By studying the thermodynamics of the phase transition we can calculate:

- The temperature at which bubble nucleation begins T_*.
- The latent heat.

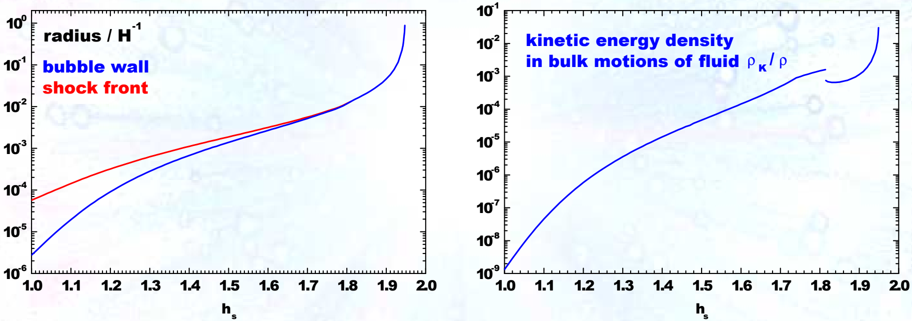
In conjunction with the hydrodynamics study, we know at any time:

- The bubble diameter as a function of its nucleation time (or temperature).
- The fluid velocity profile around the wall of each bubble.
  (Hence, the profile of kinetic energy distribution)

Assuming that turbulence *inherits* features present at collision time, the size of the biggest bubbles and the average kinetic energy density can be used to estimate the generation parameters L_S and ρ_K.

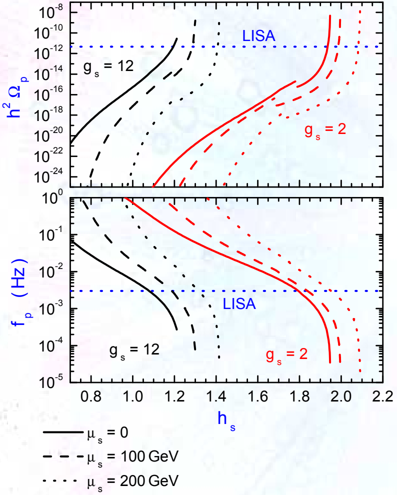
We did calculations for an extension of the *Standard Model* with strongly coupled scalar bosons to the Higgs field.

L. L., A. Megevand and A. D. Sanchez, JCAP **1210**, 024 (2012).



h_s is a coupling constant between scalars and Higgs
(It represents the strength of the interaction between these particles)

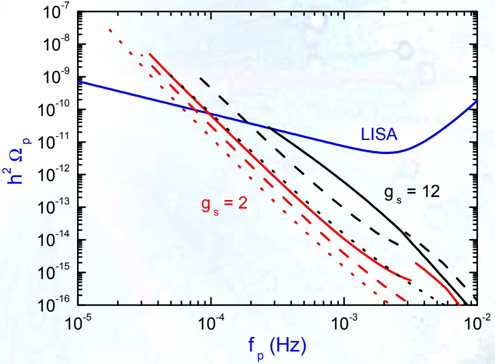
Extra coupled scalar bosons to the Higgs field.



$$h^2\Omega_{\rm GW}(f) \equiv \frac{h^2}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f}$$

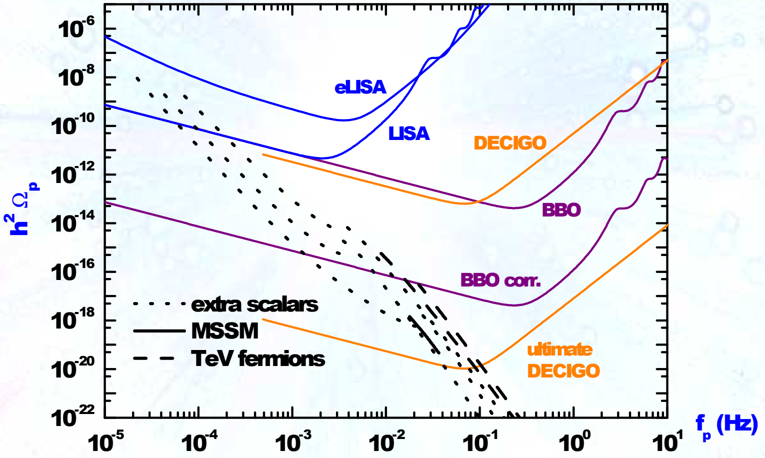
f_p = peak frequency

$h^2\Omega_{\rm p} = h^2\Omega_{\rm GW}(f_p)$ = peak intensity



Other models and detectors

It is difficult to get a signal observable by LISA,
although we have seen that the conjunction between other models and planned detectors suggests greater chances of detection.



Work in progress

The goal is to take into account the different bubble sizes present at bubble collisions (the literature assumes that the turbulence receives energy from a single length scale)

- Compute size distribution OK
- Compute turbulence (Non-trivial)
- Compute gravitational waves

Prospects

We have in mind:

- refine hydrodynamics calculations.
- consider alternative methods to detect the gravitational waves generated in cosmological phase transitions (CMB Polarization???).